On a Two-Stage Stochastic Knapsack Problem with Probabilistic Constraint

Stefanie Kosuch Abdel Lisser

Université Paris XI (LRI) France

Cologne-Twente Workshop on
Graphs and Combinatorial Optimization
June 02-04, 2009
Paris, France

1 Introduction

Outline

Two-Stage Knapsack Problems with random weights

Two-Stage Knapsack Problems with random weights

- Given a knapsack with fix capacity c

Two-Stage Knapsack Problems with random weights

- Given a knapsack with fix capacity c
- Given n items

Two-Stage Knapsack Problems with random weights

- Given a knapsack with fix capacity c
- Given n items
- χ_i: random variable representing the weight of item i

Two-Stage Knapsack Problems with random weights

- Given a knapsack with fix capacity c
- Given n items
- χ_i: random variable representing the weight of item i
- r_i: reward per weight unit of item i

Two-Stage Knapsack Problems with random weights

- Given a knapsack with fix capacity c
- Given n items
- χ_i: random variable representing the weight of item i
- r_i: reward per weight unit of item i
- First stage: items can be put in the knapsack

Two-Stage Knapsack Problems with random weights

- Given a knapsack with fix capacity c
- Given n items
- χ_i: random variable representing the weight of item i
- r_i: reward per weight unit of item i
- First stage: items can be put in the knapsack
- First stage $\longleftrightarrow$ second stage: item weights are revealed

Two-Stage Knapsack Problems with random weights

- Given a knapsack with fix capacity c
- Given n items
- χ_i: random variable representing the weight of item i
- r_i: reward per weight unit of item i
- First stage: items can be put in the knapsack
- First stage $\longleftrightarrow$ second stage: item weights are revealed
- Second stage: The decision can be corrected

Two-Stage Knapsack Problems with random weights

- Given a knapsack with fix capacity c
- Given n items
- χ_i: random variable representing the weight of item i
- r_i: reward per weight unit of item i
- First stage: items can be put in the knapsack
- First stage $\longleftrightarrow$ second stage: item weights are revealed
- Second stage: Items can be removed in case of an overweight

Two-Stage Knapsack Problems with random weights

- Given a knapsack with fix capacity c
- Given n items
- χ_i: random variable representing the weight of item i
- r_i: reward per weight unit of item i
- First stage: items can be put in the knapsack
- First stage $\longleftrightarrow$ second stage: item weights are revealed
- Second stage: Items can be removed in case of an overweight
- Correction of the decision causes penalty

Introducing a probability constraint in the first stage

Introducing a probability constraint in the first stage

- Correction of a decision might cause problems that are not "computable"

Introducing a probability constraint in the first stage

- Correction of a decision might cause problems that are not "computable"
- Restrict the need of a correction (to the worst cases)

Introducing a probability constraint in the first stage

- Correction of a decision might cause problems that are not "computable"
- Restrict the need of a correction (to the worst cases)
- $\Longrightarrow$ probability constraint in the first stage

Outline

Two-Stage Stochastic Knapsack Problem with Probabilistic Constraint

Two-Stage Stochastic Knapsack Problem with Probabilistic Constraint

$$(TSKP) \quad \max_{x \in \{0,1\}^n} \quad \mathbb{E}[\sum_{i=1}^{n} r_i \chi_i x_i]$$

s.t.

x: vector of the first stage decision variables

Two-Stage Stochastic Knapsack Problem with Probabilistic Constraint

$$(TSKP) \quad \max_{x\in\{0,1\}^n} \quad \mathbb{E}[\sum_{i=1}^{n} r_i \chi_i x_i]$$

$$\text{s.t.} \quad \mathbb{P}\{\sum_{i=1}^{n} \chi_i x_i \leq c\} \geq p$$

x: vector of the first stage decision variables

$p \in (0.5, 1]$

Two-Stage Stochastic Knapsack Problem with Probabilistic Constraint

$$(TSKP) \quad \max_{x\in\{0,1\}^n} \quad \mathbb{E}[\sum_{i=1}^{n} r_i \chi_i x_i] - d \cdot \mathbb{E}[\mathcal{Q}(x,\chi)]$$

$$\text{s.t.} \quad \mathbb{P}\{\sum_{i=1}^{n} \chi_i x_i \leq c\} \geq p$$

x: vector of the first stage decision variables

$p \in (0.5, 1]$, $d > \max_{i\in\{1,\dots,n\}} r_i$

Two-Stage Stochastic Knapsack Problem with Probabilistic Constraint

$$(TSKP) \quad \max_{x \in \{0,1\}^n} \quad \mathbb{E}[\sum_{i=1}^{n} r_i \chi_i x_i] - d \cdot \mathbb{E}[\mathcal{Q}(x,\chi)]$$

$$\text{s.t.} \quad \mathbb{P}\{\sum_{i=1}^{n} \chi_i x_i \leq c\} \geq p$$

$$\mathcal{Q}(x,\chi) = \min_{y \in \{0,1\}^n} \sum_{i=1}^{n} \chi_i y_i,$$

x: vector of the first stage decision variables

y: vector of the second stage decision variables (recourse action)

$p \in (0.5, 1]$, $d > \max_{i \in \{1,\dots,n\}} r_i$

Two-Stage Stochastic Knapsack Problem with Probabilistic Constraint

$$(TSKP) \qquad \max_{x\in\{0,1\}^n} \quad \mathbb{E}[\sum_{i=1}^{n} r_i\chi_i x_i] - d\cdot\mathbb{E}[\mathcal{Q}(x,\chi)]$$

$$\text{s.t.} \quad \mathbb{P}\{\sum_{i=1}^{n}\chi_i x_i \leq c\} \geq p$$

$$\mathcal{Q}(x,\chi) = \min_{y\in\{0,1\}^n} \sum_{i=1}^{n}\chi_i y_i,$$

$$\text{s.t.} \;\; y_k \leq x_k, \;\; k = 1,\ldots,n.$$

x: vector of the first stage decision variables
y: vector of the second stage decision variables (recourse action)
$p\in (0.5, 1]$, $d> \max_{i\in\{1,\ldots,n\}} r_i$

Two-Stage Stochastic Knapsack Problem with Probabilistic Constraint

$$(TSKP) \quad \max_{x\in\{0,1\}^n} \quad \mathbb{E}[\sum_{i=1}^{n} r_i\chi_i x_i] - d\cdot\mathbb{E}[\mathcal{Q}(x,\chi)]$$

$$\text{s.t.} \quad \mathbb{P}\{\sum_{i=1}^{n}\chi_i x_i \leq c\} \geq p$$

$$\mathcal{Q}(x,\chi) = \min_{y\in\{0,1\}^n} \sum_{i=1}^{n}\chi_i y_i,$$

$$\text{s.t.} \;\; y_k \leq x_k, \;\; k = 1,\ldots,n.$$

$$\sum_{i=1}^{n}(x_i - y_i)\chi_i \leq c.$$

x: vector of the first stage decision variables
y: vector of the second stage decision variables (recourse action)
$p \in (0.5, 1]$, $d > \max_{i\in\{1,\ldots,n\}} r_i$

Assumption

Assumption

- Continuous probability distribution

Assumption

- Continuous probability distribution
- Weights are independently normally distributed

Assumption

- Continuous probability distribution
- Weights are independently normally distributed
- Mean μ_i and standard deviation σ_i of each item i is known

Two-Stage Stochastic Knapsack Problem with Probabilistic Constraint

$$(TSKP) \quad \max_{x\in\{0,1\}^n} \quad \mathbb{E}[\sum_{i=1}^{n} r_i \chi_i x_i] - d \cdot \mathbb{E}[\mathcal{Q}(x,\chi)]$$

$$\text{s.t.} \quad \mathbb{P}\{\sum_{i=1}^{n} \chi_i x_i \leq c\} \geq p$$

$$\mathcal{Q}(x,\chi) = \min_{y\in\{0,1\}^n} \sum_{i=1}^{n} \chi_i y_i,$$

$$\text{s.t.} \quad y_k \leq x_k, \quad k = 1,\ldots,n.$$

$$\sum_{i=1}^{n} (x_i - y_i)\chi_i \leq c.$$

x: vector of the first stage decision variables

y: vector of the second stage decision variables (recourse action)

$p \in (0.5, 1]$, $d > \max_{i\in\{1,\ldots,n\}} r_i$

Problem "Solving" Method

Problem "Solving" Method

- Search First Stage solution space using Branch-and-Bound algorithm

Problem "Solving" Method

- Search First Stage solution space using Branch-and-Bound algorithm
- Compute *UB*s by solving relaxed problem

Problem "Solving" Method

- Search First Stage solution space using Branch-and-Bound algorithm
- Compute *UB*s by solving relaxed problem
- Given feasible first stage solution: Compute *UB* on Second Stage problem to get *LB* on overall problem

Outline

1. Introduction
2. Mathematical Formulation & Problem Solving
3. Calculating Upper bounds
4. Calculating Lower Bounds
   - The general case
   - The case of similar items
5. Some remarks on numerical testing
6. Conclusion

Idea

Idea

- Solve the corresponding relaxed problem

Idea

- Solve the corresponding relaxed problem
- $x \in \{0,1\}^n \longrightarrow x \in [0,1]^n$

Idea

- Solve the corresponding relaxed problem
- $x \in \{0,1\}^n \longrightarrow x \in [0,1]^n$
- $y \in \{0,1\}^n \longrightarrow y \in [0,1]^n$

Idea

- Solve the corresponding relaxed problem
- $x \in \{0,1\}^n \longrightarrow x \in [0,1]^n$
- $y \in \{0,1\}^n \longrightarrow y \in [0,1]^n$

Idea

- Solve the corresponding relaxed problem
- $x \in \{0,1\}^n \longrightarrow x \in [0,1]^n$
- $y \in \{0,1\}^n \longrightarrow y \in [0,1]^n$

Interpretation of relaxed second stage problem

Idea

- Solve the corresponding relaxed problem
- $x \in \{0,1\}^n \longrightarrow x \in [0,1]^n$
- $y \in \{0,1\}^n \longrightarrow y \in [0,1]^n$

Interpretation of relaxed second stage problem

- Remove exactly the overweight

Idea

- Solve the corresponding relaxed problem
- $x \in \{0,1\}^n \longrightarrow x \in [0,1]^n$
- $y \in \{0,1\}^n \longrightarrow y \in [0,1]^n$

Interpretation of relaxed second stage problem

- Remove exactly the overweight
- $\mathbb{E}\left[\mathcal{Q}(x,\chi) | y \in [0,1]^n\right] = \mathbb{E}\left[[\sum_{i=1}^{n} x_i\chi_i - c]^+\right]$

Idea

- Solve the corresponding relaxed problem
- $x \in \{0,1\}^n \longrightarrow x \in [0,1]^n$
- $y \in \{0,1\}^n \longrightarrow y \in [0,1]^n$

Interpretation of relaxed second stage problem

- Remove exactly the overweight
- $\mathbb{E}\left[\mathcal{Q}(x,\chi) | y \in [0,1]^n\right] = \mathbb{E}\left[[\sum_{i=1}^{n} x_i \chi_i - c]^+\right]$

Idea

- Solve the corresponding relaxed problem
- $x \in \{0,1\}^n \longrightarrow x \in [0,1]^n$
- $y \in \{0,1\}^n \longrightarrow y \in [0,1]^n$

Interpretation of relaxed second stage problem

- Remove exactly the overweight
- $\mathbb{E}\left[\mathcal{Q}(x,\chi) | y \in [0,1]^n\right] = \mathbb{E}\left[[\sum_{i=1}^n x_i \chi_i - c]^+\right]$

$[x]^+ := \max(0,x)$

Relaxed Simple Recourse Knapsack Problem

Relaxed Simple Recourse Knapsack Problem

$$(SRKP) \quad \max_{x\in[0,1]^n} \quad \mathbb{E}\left[\sum_{i=1}^{n} r_i \chi_i x_i\right] - d \cdot \mathbb{E}\left[\left[\sum x_i \chi_i - c\right]^+\right]$$

$$\text{s.t.} \quad \mathbb{P}\left\{\sum_{i=1}^{n} \chi_i x_i \leq c\right\} \geq p$$

Relaxed Simple Recourse Knapsack Problem

$$(SRKP) \quad \max_{x\in[0,1]^n} \quad \mathbb{E}\left[\sum_{i=1}^{n} r_i\chi_i x_i\right] - d\cdot\mathbb{E}\left[\left[\sum x_i\chi_i - c\right]^+\right]$$

$$\text{s.t.} \quad \mathbb{P}\left\{\sum_{i=1}^{n}\chi_i x_i \leq c\right\} \geq p$$

Properties

Relaxed Simple Recourse Knapsack Problem

$$(SRKP) \quad \max_{x\in[0,1]^n} \quad \mathbb{E}\left[\sum_{i=1}^{n} r_i \chi_i x_i\right] - d \cdot \mathbb{E}\left[\left[\sum x_i \chi_i - c\right]^+\right]$$

$$\text{s.t.} \quad \mathbb{P}\left\{\sum_{i=1}^{n} \chi_i x_i \leq c\right\} \geq p$$

Properties

- Deterministic equivalent objective function

Relaxed Simple Recourse Knapsack Problem

$$(SRKP) \quad \max_{x\in[0,1]^n} \quad \mathbb{E}\left[\sum_{i=1}^{n} r_i \chi_i x_i\right] - d \cdot \mathbb{E}\left[\left[\sum x_i \chi_i - c\right]^+\right]$$

$$\text{s.t.} \quad \mathbb{P}\left\{\sum_{i=1}^{n} \chi_i x_i \leq c\right\} \geq p$$

Properties

- Deterministic equivalent objective function
- $\Rightarrow$ Exact evaluation possible

Relaxed Simple Recourse Knapsack Problem

$$(SRKP) \quad \max_{x\in[0,1]^n} \quad \mathbb{E}\left[\sum_{i=1}^{n} r_i \chi_i x_i\right] - d\cdot\mathbb{E}\left[\left[\sum x_i\chi_i - c\right]^+\right]$$

$$\text{s.t.} \quad \mathbb{P}\left\{\sum_{i=1}^{n} \chi_i x_i \leq c\right\} \geq p$$

Properties

- Deterministic equivalent objective function
- $\Rightarrow$ Exact evaluation possible
- Concave objective function

Relaxed Simple Recourse Knapsack Problem

$$(SRKP) \quad \max_{x\in[0,1]^n} \quad \mathbb{E}\left[\sum_{i=1}^{n} r_i \chi_i x_i\right] - d \cdot \mathbb{E}\left[\left[\sum x_i \chi_i - c\right]^+\right]$$

$$\text{s.t.} \quad \mathbb{P}\left\{\sum_{i=1}^{n} \chi_i x_i \leq c\right\} \geq p$$

Properties

- Deterministic equivalent objective function
- $\Rightarrow$ Exact evaluation possible
- Concave objective function
- Convex feasible set

Simple Recourse Knapsack Problem

$$(SRKP) \quad \max_{x\in\{0,1\}^n} \quad \mathbb{E}\left[\sum_{i=1}^{n} r_i \chi_i x_i\right] - d \cdot \mathbb{E}\left[[\sum x_i \chi_i - c]^+\right]$$

$$\text{s.t.} \quad \mathbb{P}\left\{\sum_{i=1}^{n} \chi_i x_i \leq c\right\} \geq p$$

Simple Recourse Knapsack Problem

$$(SRKP) \quad \max_{x\in\{0,1\}^n} \quad \mathbb{E}\left[\sum_{i=1}^{n} r_i\chi_i x_i\right] - d\cdot\mathbb{E}\left[[\sum x_i\chi_i - c]^+\right]$$

$$\text{s.t.} \quad \mathbb{P}\left\{\sum_{i=1}^{n}\chi_i x_i \le c\right\} \ge p$$

Problem Solving method[a]

[a] *Upper bounds for the 0-1 stoch. knaps. prob. and a B&B algorithm*
Accepted for publication in *Annals of Operations Research*

Simple Recourse Knapsack Problem

$$(SRKP) \quad \max_{x\in\{0,1\}^n} \quad \mathbb{E}\left[\sum_{i=1}^{n} r_i \chi_i x_i\right] - d \cdot \mathbb{E}\left[[\sum x_i \chi_i - c]^+\right]$$

$$\text{s.t.} \quad \mathbb{P}\left\{\sum_{i=1}^{n} \chi_i x_i \le c\right\} \ge p$$

Problem Solving method[a]

[a] *Upper bounds for the 0-1 stoch. knaps. prob. and a B&B algorithm* Accepted for publication in *Annals of Operations Research*

- Solve Lagrangian MinMax Problem

Simple Recourse Knapsack Problem

$$(SRKP) \quad \max_{x \in \{0,1\}^n} \quad \mathbb{E}\left[\sum_{i=1}^{n} r_i \chi_i x_i\right] - d \cdot \mathbb{E}\left[[\sum x_i \chi_i - c]^+\right]$$

$$\text{s.t.} \quad \mathbb{E}\left[\mathbb{1}_{\mathbb{R}^+}(c - \sum_{i=1}^{n} \chi_i x_i)\right] \geq p$$

Problem Solving method[a]

[a] *Upper bounds for the 0-1 stoch. knaps. prob. and a B&B algorithm* Accepted for publication in *Annals of Operations Research*

- Solve Lagrangian MinMax Problem

Simple Recourse Knapsack Problem

$$(SRKP) \quad \max_{x \in \{0,1\}^n} \quad \mathbb{E}\left[\sum_{i=1}^{n} r_i \chi_i x_i\right] - d \cdot \mathbb{E}\left[\left[\sum x_i \chi_i - c\right]^+\right]$$

$$\text{s.t.} \quad \mathbb{E}\left[\mathbb{1}_{\mathbb{R}^+}\left(c - \sum_{i=1}^{n} \chi_i x_i\right)\right] \geq p$$

Problem Solving method[a]

[a] *Upper bounds for the 0-1 stoch. knaps. prob. and a B&B algorithm*
Accepted for publication in *Annals of Operations Research*

- Solve Lagrangian MinMax Problem
- Stochastic gradient type algorithm

Outline

Problem

Problem

Given a first stage solution $\tilde{x}$, compute an upper bound on the expectation of the corresponding second stage solution $\mathbb{E}[Q(\tilde{x},\chi)]$ in order to get a lower bound on the overall solution.

Problem

Given a first stage solution $\widetilde{x}$, compute an upper bound on the expectation of the corresponding second stage solution $\mathbb{E}[Q(\widetilde{x}, \chi)]$ in order to get a lower bound on the overall solution.

Idea

Problem

Given a first stage solution $\tilde{x}$, compute an upper bound on the expectation of the corresponding second stage solution $\mathbb{E}[Q(\tilde{x}, \chi)]$ in order to get a lower bound on the overall solution.

Idea

Split the expectation of the second stage solution as follows:

$$\mathbb{E}[\mathcal{Q}(\tilde{x}, \chi)] = \mathbb{E}\left[[\sum_{i=1}^{n} \tilde{x}_i \chi_i - c]^+ \right] + \mathbb{E}\left[\sum_{i=1}^{n} y_i^* \chi_i - [\sum_{i=1}^{n} \tilde{x}_i \chi_i - c]^+ \right]$$

Problem

Given a first stage solution $\tilde{x}$, compute an upper bound on the expectation of the corresponding second stage solution $\mathbb{E}[Q(\tilde{x}, \chi)]$ in order to get a lower bound on the overall solution.

Idea

Split the expectation of the second stage solution as follows:

$$\mathbb{E}[\mathcal{Q}(\tilde{x}, \chi)] = \mathbb{E}\left[[\sum_{i=1}^{n} \tilde{x}_i \chi_i - c]^+ \right] + \mathbb{E}\left[\sum_{i=1}^{n} y_i^* \chi_i - [\sum_{i=1}^{n} \tilde{x}_i \chi_i - c]^+ \right]$$

$y^* = y^*(\tilde{x})$ corresponding optimal second stage solution

Problem

Given a first stage solution $\tilde{x}$, compute an upper bound on the expectation of the corresponding second stage solution $\mathbb{E}[Q(\tilde{x}, \chi)]$ in order to get a lower bound on the overall solution.

Idea

Split the expectation of the second stage solution as follows:

$$\mathbb{E}[\mathcal{Q}(\tilde{x}, \chi)] = \mathbb{E}\left[[\sum_{i=1}^{n} \tilde{x}_i \chi_i - c]^+\right] + \mathbb{E}\left[\sum_{i=1}^{n} y_i^* \chi_i - [\sum_{i=1}^{n} \tilde{x}_i \chi_i - c]^+\right]$$

$y^* = y^*(\tilde{x})$ corresponding optimal second stage solution

Interpretation

Interpretation

* $\hat{\chi}$: realization of χ:

Interpretation

- $\hat{\chi}$: realization of χ:

$$\sum_{i=1}^{n} y_i^* \hat{\chi}_i - [\sum_{i=1}^{n} \tilde{x}_i \hat{\chi}_i - c]^+$$

$$=$$

Amount of weight removed from the knapsack **in addition** to the overweight

Interpretation

Amount of weight removed from the knapsack **in addition** to the overweight

Simple bounds

Interpretation

Amount of weight removed from the knapsack **in addition** to the overweight

Simple bounds

If $\tilde{x}_i \hat{\chi}_i \leq c$:

Interpretation

Amount of weight removed from the knapsack **in addition** to the overweight

Simple bounds

$$\text{If } \tilde{x}_i \hat{\chi}_i \leq c: \quad \sum_{i=1}^{n} y_i^* \hat{\chi}_i - [\sum_{i=1}^{n} \tilde{x}_i \hat{\chi}_i - c]^+ = 0$$

Interpretation

Amount of weight removed from the knapsack **in addition** to the overweight

Simple bounds

If $\tilde{x}_i \hat{\chi}_i \leq c$: $$\sum_{i=1}^{n} y_i^* \hat{\chi}_i - [\sum_{i=1}^{n} \tilde{x}_i \hat{\chi}_i - c]^+ = 0$$

else:

Interpretation

Amount of weight removed from the knapsack **in addition** to the overweight

Simple bounds

$$\text{If } \tilde{x}_i \hat{\chi}_i \leq c: \quad \sum_{i=1}^{n} y_i^* \hat{\chi}_i - [\sum_{i=1}^{n} \tilde{x}_i \hat{\chi}_i - c]^+ = 0$$

$$\text{else:} \quad \sum_{i=1}^{n} y_i^* \hat{\chi}_i - [\sum_{i=1}^{n} \tilde{x}_i \hat{\chi}_i - c]^+ < \max_{i \in S} \hat{\chi}_i$$

$\mathbf{S} := \{\mathbf{i} \in \{\mathbf{1}, \dots, \mathbf{n}\} : \mathbf{x_i} = \mathbf{1}\}$

Interpretation

Amount of weight removed from the knapsack **in addition** to the overweight

Simple bounds

$$\text{If } \tilde{x}_i \hat{\chi}_i \leq c: \quad \sum_{i=1}^{n} y_i^* \hat{\chi}_i - [\sum_{i=1}^{n} \tilde{x}_i \hat{\chi}_i - c]^+ = 0$$

$$\text{else:} \quad \sum_{i=1}^{n} y_i^* \hat{\chi}_i - [\sum_{i=1}^{n} \tilde{x}_i \hat{\chi}_i - c]^+ < \max_{i \in S} \hat{\chi}_i$$

$\mathbf{S := \{i \in \{1, \ldots, n\} : x_i = 1\}}$

Interpretation

Amount of weight removed from the knapsack **in addition** to the overweight

Simple bounds

$$\text{If } \tilde{x}_i \hat{\chi}_i \leq c: \quad \sum_{i=1}^{n} y_i^* \hat{\chi}_i - [\sum_{i=1}^{n} \tilde{x}_i \hat{\chi}_i - c]^+ = 0$$

$$\text{else:} \quad \sum_{i=1}^{n} y_i^* \hat{\chi}_i - [\sum_{i=1}^{n} \tilde{x}_i \hat{\chi}_i - c]^+ < \max_{i \in S} \hat{\chi}_i$$

$$\mathbf{S} := \{\mathbf{i} \in \{\mathbf{1}, \ldots, \mathbf{n}\} : \mathbf{x_i} = \mathbf{1}\}$$

$$\Rightarrow \sum_{i=1}^{n} y_i^* \hat{\chi}_i - [\sum_{i=1}^{n} \tilde{x}_i \hat{\chi}_i - c]^+ \leq \mathbb{1}_{\mathbb{R}+}(\sum_{i=1}^{n} \tilde{x}_i \chi_i - c) \max_{i \in S} \hat{\chi}_i$$

Upper bound on the expected second stage solution

Upper bound on the expected second stage solution

$$\mathbb{E}[\mathcal{Q}(\tilde{x}, \chi)] =$$

Upper bound on the expected second stage solution

$$\mathbb{E}[\mathcal{Q}(\tilde{x},\chi)] = \mathbb{E}\left[[\sum_{i=1}^{n} \tilde{x}_i \chi_i - c]^+\right] + \mathbb{E}\left[\sum_{i=1}^{n} y_i^* \chi_i - [\sum_{i=1}^{n} \tilde{x}_i \chi_i - c]^+\right]$$

Upper bound on the expected second stage solution

$$\mathbb{E}[\mathcal{Q}(\tilde{x},\chi)] = \mathbb{E}\left[[\sum_{i=1}^{n}\tilde{x}_i\chi_i - c]^+\right] + \mathbb{E}\left[\sum_{i=1}^{n} y_i^*\chi_i - [\sum_{i=1}^{n}\tilde{x}_i\chi_i - c]^+\right]$$

$$\leq \mathbb{E}\left[[\sum_{i=1}^{n}\tilde{x}_i\chi_i - c]^+\right] + \mathbb{E}\left[\mathbb{1}_{\mathbb{R}^+}(\sum_{i=1}^{n}\tilde{x}_i\chi_i - c)\max_{i\in S}\chi_i\right]$$

Bounding $\mathbb{E}\left[\max_{i \in S} \chi_i\right]$

- $\chi^S_{max} := \max_{i \in S} \chi_i$

Bounding $\mathbb{E}\left[\max_{i \in S} \chi_i\right]$

- $\chi_{max}^{S} := \max_{i \in S} \chi_i$
- CDF: $\Phi_{max}^{S} = \prod_{i \in S} \Phi_i$

Bounding $\mathbb{E}\left[\max_{i \in S} \chi_i\right]$

- $\chi^S_{max} := \max_{i \in S} \chi_i$
- CDF: $\Phi^S_{max} = \prod_{i \in S} \Phi_i$

$$\mathbb{E}\left[\chi^S_{max}\right] = \int_{-\infty}^{\infty} \chi^S_{max}\, \varphi^S_{max}(\chi^S_{max})\, d\chi^S_{max}$$

Bounding $\mathbb{E}\left[\max_{i \in S} \chi_i\right]$

- $\chi^S_{max} := \max_{i \in S} \chi_i$
- CDF: $\Phi^S_{max} = \prod_{i \in S} \Phi_i$
- "practically": $\exists \beta$ s.t. $\mathbb{P}\{\chi^S_{max} > \beta\} = 0$

$$\mathbb{E}\left[\chi^S_{max}\right] = \int_{-\infty}^{\infty} \chi^S_{max}\, \varphi^S_{max}(\chi^S_{max})\, d\chi^S_{max}$$

Bounding $\mathbb{E}\left[\max_{i \in S} \chi_i\right]$

- $\chi^S_{max} := \max_{i \in S} \chi_i$
- CDF: $\Phi^S_{max} = \prod_{i \in S} \Phi_i$
- "practically": $\exists \beta$ s.t. $\mathbb{P}\{\chi^S_{max} > \beta\} = 0$

$$\mathbb{E}\left[\chi^S_{max}\right] = \int_{-\infty}^{\beta} \chi^S_{max}\, \varphi^S_{max}(\chi^S_{max})\, d\chi^S_{max}$$

Bounding $\mathbb{E}\left[\max_{i \in S} \chi_i\right]$

- $\chi_{max}^S := \max_{i \in S} \chi_i$
- CDF: $\Phi_{max}^S = \prod_{i \in S} \Phi_i$
- "practically": $\exists \beta$ s.t. $\mathbb{P}\{\chi_{max}^S > \beta\} = 0$
- $\Rightarrow$ Divide interval $(-\infty, \beta]$ in disjunct intervals $(\alpha_k, \beta_k]$, $k = 1, \ldots, K$

$$\mathbb{E}\left[\chi_{max}^S\right] = \int_{-\infty}^{\beta} \chi_{max}^S \, \varphi_{max}^S(\chi_{max}^S) \, d\chi_{max}^S$$

Bounding $\mathbb{E}\left[\max_{i \in S} \chi_i\right]$

- $\chi^S_{max} := \max_{i \in S} \chi_i$
- CDF: $\Phi^S_{max} = \prod_{i \in S} \Phi_i$
- "practically": $\exists \beta$ s.t. $\mathbb{P}\{\chi^S_{max} > \beta\} = 0$
- $\Rightarrow$ Divide interval $(-\infty, \beta]$ in disjunct intervals $(\alpha_k, \beta_k]$, $k = 1, \ldots, K$

$$\mathbb{E}\left[\chi^S_{max}\right] = \int_{-\infty}^{\beta} \chi^S_{max}\, \varphi^S_{max}(\chi^S_{max})\, d\chi^S_{max}$$
$$= \sum_{k=1}^{K} \int_{\alpha_k}^{\beta_k} \chi^S_{max} \varphi^S_{max}(\chi^S_{max})\, d\chi^S_{max}$$

Bounding $\mathbb{E}\left[\max_{i\in S}\chi_i\right]$

- $\chi^S_{max} := \max_{i\in S}\chi_i$
- CDF: $\Phi^S_{max} = \prod_{i\in S}\Phi_i$
- "practically": $\exists\beta$ s.t. $\mathbb{P}\{\chi^S_{max} > \beta\} = 0$
- $\Rightarrow$ Divide interval $(-\infty, \beta]$ in disjunct intervals $(\alpha_k, \beta_k]$, $k = 1, \ldots, K$

$$\begin{aligned}\mathbb{E}\left[\chi^S_{max}\right] &= \int_{-\infty}^{\beta} \chi^S_{max}\,\varphi^S_{max}(\chi^S_{max})\,d\chi^S_{max}\\ &= \sum_{k=1}^{K}\int_{\alpha_k}^{\beta_k} \chi^S_{max}\varphi^S_{max}(\chi^S_{max})\,d\chi^S_{max}\\ &\leq \sum_{k=1}^{K}\beta_k\int_{\alpha_k}^{\beta_k} \varphi^S_{max}(\chi^S_{max})\,d\chi^S_{max}\end{aligned}$$

Bounding $\mathbb{E}\left[\max_{i \in S} \chi_i\right]$

- $\chi^S_{max} := \max_{i \in S} \chi_i$
- CDF: $\Phi^S_{max} = \prod_{i \in S} \Phi_i$
- "practically": $\exists \beta$ s.t. $\mathbb{P}\{\chi^S_{max} > \beta\} = 0$
- $\Rightarrow$ Divide interval $(-\infty, \beta]$ in disjunct intervals $(\alpha_k, \beta_k]$, $k = 1, \dots, K$

$$\begin{aligned}
\mathbb{E}\left[\chi^S_{max}\right] &= \int_{-\infty}^{\beta} \chi^S_{max}\, \varphi^S_{max}(\chi^S_{max})\, d\chi^S_{max} \\
&= \sum_{k=1}^{K} \int_{\alpha_k}^{\beta_k} \chi^S_{max} \varphi^S_{max}(\chi^S_{max})\, d\chi^S_{max} \\
&\leq \sum_{k=1}^{K} \beta_k (\Phi^S_{max}(\beta_k) - \Phi^S_{max}(\alpha_k))
\end{aligned}$$

Bounding $\mathbb{E}\left[\mathbb{1}_{\mathbb{R}^+}(\sum_{i=1}^{n} \tilde{x}_i \chi_i - c) \max_{i \in S} \chi_i\right]$

- $\mathbb{1}_{\mathbb{R}^+}(\sum_{i=1}^{n} \tilde{x}_i \hat{\chi}_i - c) = 1 \implies \hat{\chi}^S_{max} \geq c/|S|$

Bounding $\mathbb{E}\left[\mathbb{1}_{\mathbb{R}^+}(\sum_{i=1}^{n} \tilde{x}_i \chi_i - c) \max_{i \in S} \chi_i\right]$

- $\mathbb{1}_{\mathbb{R}^+}(\sum_{i=1}^{n} \tilde{x}_i \hat{\chi}_i - c) = 1 \implies \hat{\chi}^S_{max} \geq c/|S|$
- $\alpha := c/|S|$

Bounding $\mathbb{E}\left[\mathbb{1}_{\mathbb{R}^+}(\sum_{i=1}^n \tilde{x}_i \chi_i - c) \max_{i \in S} \chi_i\right]$

- $\mathbb{1}_{\mathbb{R}^+}(\sum_{i=1}^n \tilde{x}_i \hat{\chi}_i - c) = 1 \implies \hat{\chi}^S_{max} \geq c/|S|$
- $\alpha := c/|S|$
- $\Rightarrow$ Divide interval $(\alpha, \beta]$ in disjunct intervals $(\widetilde{\alpha}_k, \widetilde{\beta}_k]$, $k = 1, \ldots, \widetilde{K}$

Bounding $\mathbb{E}\left[\mathbb{1}_{\mathbb{R}^+}(\sum_{i=1}^n \tilde{x}_i \chi_i - c) \max_{i \in S} \chi_i\right]$

- $\mathbb{1}_{\mathbb{R}^+}(\sum_{i=1}^n \tilde{x}_i \hat{\chi}_i - c) = 1 \implies \hat{\chi}^S_{max} \geq c/|S|$
- $\alpha := c/|S|$
- $\Rightarrow$ Divide interval $(\alpha, \beta]$ in disjunct intervals $(\widetilde{\alpha}_k, \widetilde{\beta}_k]$, $k = 1, \ldots, \widetilde{K}$

Bounding $\mathbb{E}\left[\mathbb{1}_{\mathbb{R}^+}(\sum_{i=1}^n \tilde{x}_i\chi_i - c)\max_{i\in S}\chi_i\right]$

- $\mathbb{1}_{\mathbb{R}^+}(\sum_{i=1}^n \tilde{x}_i\hat{\chi}_i - c) = 1 \implies \hat{\chi}^S_{max} \geq c/|S|$
- $\alpha := c/|S|$
- $\Rightarrow$ Divide interval $(\alpha, \beta]$ in disjunct intervals $(\widetilde{\alpha}_k, \widetilde{\beta}_k]$, $k = 1, \ldots, \widetilde{K}$

$$\mathbb{E}\left[\mathbb{1}_{\mathbb{R}^+}(\sum_{i=1}^n \tilde{x}_i\chi_i - c)\chi^S_{max}\right] \leq \sum_{k=1}^{\widetilde{K}} \widetilde{\beta}_k \mathbb{P}\{\chi^S_{max} \in (\widetilde{\alpha}_k, \widetilde{\beta}_k]\}$$

$$= \sum_{k=1}^{\widetilde{K}} \widetilde{\beta}_k(\Phi^S_{max}(\widetilde{\beta}_k) - \Phi^S_{max}(\widetilde{\alpha}_k))$$

Upper bound on expected second stage solution II

Upper bound on expected second stage solution II

$$\mathbb{E}[\mathcal{Q}(\tilde{x},\chi)] = \mathbb{E}\left[[\sum_{i=1}^{n}\tilde{x}_i\chi_i - c]^+\right] + \mathbb{E}\left[\sum_{i=1}^{n} y_i^*\chi_i - [\sum_{i=1}^{n}\tilde{x}_i\chi_i - c]^+\right]$$

$$\leq \mathbb{E}\left[[\sum_{i=1}^{n}\tilde{x}_i\chi_i - c]^+\right] + \mathbb{E}\left[\mathbb{1}_{\mathbb{R}^+}(\sum_{i=1}^{n}\tilde{x}_i\chi_i - c)\max_{i\in S}\chi_i\right]$$

Upper bound on expected second stage solution II

$$\mathbb{E}[\mathcal{Q}(\tilde{x},\chi)] = \mathbb{E}\left[[\sum_{i=1}^{n}\tilde{x}_i\chi_i - c]^+\right] + \mathbb{E}\left[\sum_{i=1}^{n} y_i^*\chi_i - [\sum_{i=1}^{n}\tilde{x}_i\chi_i - c]^+\right]$$

$$\leq \mathbb{E}\left[[\sum_{i=1}^{n}\tilde{x}_i\chi_i - c]^+\right] + \mathbb{E}\left[\mathbb{1}_{\mathbb{R}^+}(\sum_{i=1}^{n}\tilde{x}_i\chi_i - c)\max_{i\in S}\chi_i\right]$$

$$\leq \mathbb{E}\left[[\sum_{i=1}^{n}\tilde{x}_i\chi_i - c]^+\right] + \sum_{k=1}^{\widetilde{K}}\widetilde{\beta}_k(\Phi_{max}^S(\widetilde{\beta}_k) - \Phi_{max}^S(\widetilde{\alpha}_k))$$

"Similar" items

"Similar" items

- "Similar" $\hat{=} \left[\frac{\max_{i,j \in \{1,\ldots,n\}} |\mu_i - \mu_j|}{\min_{i \in \{1,\ldots,n\}} \mu_i} << 1/2 \right]$

"Similar" items

- "Similar" $\triangleq \left[\frac{\max_{i,j\in\{1,\dots,n\}} |\mu_i - \mu_j|}{\min_{i\in\{1,\dots,n\}} \mu_i} << 1/2\right]$
- $\implies \mathbb{P}\{\exists i,j : \chi_i > 2\chi_j\} \approx 0$

"Similar" items

- "Similar" $\hat{=} \left[\frac{\max_{i,j \in \{1,\ldots,n\}} |\mu_i - \mu_j|}{\min_{i \in \{1,\ldots,n\}} \mu_i} << 1/2 \right]$

- $\Longrightarrow \mathbb{P}\{\exists i,j : \chi_i > 2\chi_j\} \approx 0$

"Similar" items

- "Similar" $\hat{=} \left[\frac{\max_{i,j \in \{1,\dots,n\}} |\mu_i - \mu_j|}{\min_{i \in \{1,\dots,n\}} \mu_i} << 1/2 \right]$
- $\Longrightarrow \mathbb{P}\{\exists i, j : \chi_i > 2\chi_j\} \approx 0$

Lemma

"Similar" items

- "Similar" $\hat{=} \left[\frac{\max_{i,j \in \{1,\dots,n\}} |\mu_i - \mu_j|}{\min_{i \in \{1,\dots,n\}} \mu_i} << 1/2 \right]$
- $\Longrightarrow \mathbb{P}\{\exists i,j : \chi_i > 2\chi_j\} \approx 0$

Lemma

If $\mathbb{P}\{\exists i,j : \chi_i > 2\chi_j\} = 0$, *it follows*

$$\sum_{i=1}^{n} y_i^* \hat{\chi}_i - [\sum_{i=1}^{n} \tilde{x}_i \hat{\chi}_i - c]^+ \leq \min_{i \in S} \hat{\chi}_i$$

Upper bound on the expected second stage solution (similar items)

Upper bound on the expected second stage solution (similar items)

$$\mathbb{E}[\mathcal{Q}(\tilde{x}, \chi)] =$$

Upper bound on the expected second stage solution (similar items)

$$\mathbb{E}[\mathcal{Q}(\tilde{x},\chi)] = \mathbb{E}\left[[\sum_{i=1}^{n}\tilde{x}_i\chi_i - c]^+\right] + \mathbb{E}\left[\sum_{i=1}^{n} y_i^*\chi_i - [\sum_{i=1}^{n}\tilde{x}_i\chi_i - c]^+\right]$$

Upper bound on the expected second stage solution (similar items)

$$\begin{aligned}\mathbb{E}[\mathcal{Q}(\tilde{x},\chi)] &= \mathbb{E}\left[[\sum_{i=1}^{n}\tilde{x}_i\chi_i - c]^+\right] + \mathbb{E}\left[\sum_{i=1}^{n} y_i^*\chi_i - [\sum_{i=1}^{n}\tilde{x}_i\chi_i - c]^+\right] \\ &\leq \mathbb{E}\left[[\sum_{i=1}^{n}\tilde{x}_i\chi_i - c]^+\right] + \mathbb{E}\left[\mathbb{1}_{\mathbb{R}^+}(\sum_{i=1}^{n}\tilde{x}_i\chi_i - c)\min_{i\in S}\chi_i\right]\end{aligned}$$

Upper bound

Upper bound

$$\min_{i \in S} \chi_i \leq \frac{\sum_{i=1}^{n} \tilde{x}_i \hat{\chi}_i}{|S|}$$

Upper bound

$$\min_{i \in S} \chi_i \leq \frac{\sum_{i=1}^{n} \tilde{x}_i \hat{\chi}_i}{|S|}$$

Upper bound on the expected second stage solution (similar items) II

Upper bound

$$\min_{i \in S} \chi_i \leq \frac{\sum_{i=1}^{n} \tilde{x}_i \hat{\chi}_i}{|S|}$$

Upper bound on the expected second stage solution (similar items) II

$$\mathbb{E}[\mathcal{Q}(\tilde{x}, \chi)] =$$

Upper bound

$$\min_{i \in S} \chi_i \leq \frac{\sum_{i=1}^n \tilde{x}_i \hat{\chi}_i}{|S|}$$

Upper bound on the expected second stage solution (similar items) II

$$\begin{aligned}\mathbb{E}[\mathcal{Q}(\tilde{x}, \chi)] &= \mathbb{E}\left[[\sum_{i=1}^n \tilde{x}_i \chi_i - c]^+ \right] + \mathbb{E}\left[\sum_{i=1}^n y_i^* \chi_i - [\sum_{i=1}^n \tilde{x}_i \chi_i - c]^+ \right] \\ &\leq \mathbb{E}\left[[\sum_{i=1}^n \tilde{x}_i \chi_i - c]^+ \right] + \mathbb{E}\left[\mathbb{1}_{\mathbb{R}^+}(\sum_{i=1}^n \tilde{x}_i \chi_i - c) \min_{i \in S} \chi_i \right]\end{aligned}$$

Upper bound

$$\min_{i \in S} \chi_i \leq \frac{\sum_{i=1}^{n} \tilde{x}_i \hat{\chi}_i}{|S|}$$

Upper bound on the expected second stage solution (similar items) II

$$\begin{aligned}
\mathbb{E}[\mathcal{Q}(\tilde{x}, \chi)] &= \mathbb{E}\left[[\sum_{i=1}^{n} \tilde{x}_i \chi_i - c]^+ \right] + \mathbb{E}\left[\sum_{i=1}^{n} y_i^* \chi_i - [\sum_{i=1}^{n} \tilde{x}_i \chi_i - c]^+ \right] \\
&\leq \mathbb{E}\left[[\sum_{i=1}^{n} \tilde{x}_i \chi_i - c]^+ \right] + \mathbb{E}\left[\mathbb{1}_{\mathbb{R}^+}(\sum_{i=1}^{n} \tilde{x}_i \chi_i - c) \min_{i \in S} \chi_i \right] \\
&\leq \mathbb{E}\left[[\sum_{i=1}^{n} \tilde{x}_i \chi_i - c]^+ \right] + \mathbb{E}\left[\mathbb{1}_{\mathbb{R}^+}(\sum_{i=1}^{n} \tilde{x}_i \chi_i - c) \frac{\sum_{i=1}^{n} \tilde{x}_i \chi_i}{|S|} \right]
\end{aligned}$$

Upper bound on the expected second stage solution (similar items) III

$$\mathbb{E}[\mathcal{Q}(\tilde{x},\chi)] \leq \mathbb{E}\left[[\sum_{i=1}^{n}\tilde{x}_i\chi_i - c]^+\right] + \mathbb{E}\left[\mathbb{1}_{\mathbb{R}^+}(\sum_{i=1}^{n}\tilde{x}_i\chi_i - c)\frac{\sum_{i=1}^{n}\tilde{x}_i\chi_i}{|S|}\right]$$

Upper bound on the expected second stage solution (similar items) III

$$\mathbb{E}[\mathcal{Q}(\tilde{x},\chi)] \leq \mathbb{E}\left[[\sum_{i=1}^{n}\tilde{x}_i\chi_i - c]^+\right] + \mathbb{E}\left[\mathbb{1}_{\mathbb{R}^+}(\sum_{i=1}^{n}\tilde{x}_i\chi_i - c)\frac{\sum_{i=1}^{n}\tilde{x}_i\chi_i}{|S|}\right]$$

$$= \mathbb{E}\left[[\sum_{i=1}^{n}\tilde{x}_i\chi_i - c]^+\right] +$$

$$\mathbb{E}\left[\frac{[\sum_{i=1}^{n}\tilde{x}_i\chi_i - c]^+}{|S|}\right] + \mathbb{E}\left[\mathbb{1}_{\mathbb{R}^+}(\sum_{i=1}^{n}\tilde{x}_i\chi_i - c)\right]\frac{c}{|S|}$$

Upper bound on the expected second stage solution (similar items) III

$$\mathbb{E}[\mathcal{Q}(\tilde{x},\chi)] \leq \mathbb{E}\left[[\sum_{i=1}^{n}\tilde{x}_i\chi_i - c]^+\right] + \mathbb{E}\left[\mathbb{1}_{\mathbb{R}^+}(\sum_{i=1}^{n}\tilde{x}_i\chi_i - c)\frac{\sum_{i=1}^{n}\tilde{x}_i\chi_i}{|S|}\right]$$

$$= \mathbb{E}\left[[\sum_{i=1}^{n}\tilde{x}_i\chi_i - c]^+\right] +$$

$$\mathbb{E}\left[\frac{[\sum_{i=1}^{n}\tilde{x}_i\chi_i - c]^+}{|S|}\right] + \mathbb{E}\left[\mathbb{1}_{\mathbb{R}^+}(\sum_{i=1}^{n}\tilde{x}_i\chi_i - c)\right]\frac{c}{|S|}$$

Upper bound on the expected second stage solution (similar items) III

$$\mathbb{E}[\mathcal{Q}(\tilde{x},\chi)] \leq \mathbb{E}\left[[\sum_{i=1}^{n}\tilde{x}_i\chi_i - c]^+\right] + \mathbb{E}\left[\mathbb{1}_{\mathbb{R}^+}(\sum_{i=1}^{n}\tilde{x}_i\chi_i - c)\frac{\sum_{i=1}^{n}\tilde{x}_i\chi_i}{|S|}\right]$$

$$= \mathbb{E}\left[[\sum_{i=1}^{n}\tilde{x}_i\chi_i - c]^+\right] +$$

$$\mathbb{E}\left[\frac{[\sum_{i=1}^{n}\tilde{x}_i\chi_i - c]^+}{|S|}\right] + \mathbb{P}\{\sum_{i=1}^{n}\tilde{x}_i\chi_i \geq c\}\frac{c}{|S|}$$

Upper bound on the expected second stage solution (similar items) III

$$\mathbb{E}[\mathcal{Q}(\tilde{x},\chi)] \leq \mathbb{E}\left[[\sum_{i=1}^{n}\tilde{x}_i\chi_i - c]^+\right] + \mathbb{E}\left[\mathbb{1}_{\mathbb{R}^+}(\sum_{i=1}^{n}\tilde{x}_i\chi_i - c)\frac{\sum_{i=1}^{n}\tilde{x}_i\chi_i}{|S|}\right]$$

$$= \mathbb{E}\left[[\sum_{i=1}^{n}\tilde{x}_i\chi_i - c]^+\right] +$$

$$\mathbb{E}\left[\frac{[\sum_{i=1}^{n}\tilde{x}_i\chi_i - c]^+}{|S|}\right] + \mathbb{P}\{\sum_{i=1}^{n}\tilde{x}_i\chi_i \geq c\}\frac{c}{|S|}$$

$$\leq (1+\frac{1}{|S|})\cdot\mathbb{E}\left[[\sum_{i=1}^{n}\tilde{x}_i\chi_i - c]^+\right] + \frac{(1-p)\cdot c}{|S|}$$

Outline

Remarks

Remarks

- No test instances for comparison available
  $\longrightarrow$ randomly generated

Remarks

- No test instances for comparison available
  $\longrightarrow$ randomly generated
- Exact solutions unknown

Remarks

- No test instances for comparison available
  $\longrightarrow$ randomly generated
- Exact solutions unknown

Remarks

- No test instances for comparison available
  $\longrightarrow$ randomly generated
- Exact solutions unknown

Numerical Results

Remarks

- No test instances for comparison available
  $\longrightarrow$ randomly generated
- Exact solutions unknown

Numerical Results

- Solving the relaxed problem: $n = 15 : \sim 3ms$, $n = 4500 : < 1s$

Remarks

- No test instances for comparison available
  $\longrightarrow$ randomly generated
- Exact solutions unknown

Numerical Results

- Solving the relaxed problem: $n = 15 : \sim 3ms$, $n = 4500 : < 1s$
- B&B-algorithm (similar items):

Remarks

- No test instances for comparison available
  $\longrightarrow$ randomly generated
- Exact solutions unknown

Numerical Results

- Solving the relaxed problem: $n = 15 : \sim 3ms$, $n = 4500 : < 1s$
- B&B-algorithm (similar items):

n	nods considered	CPU (sec)	Gap
15	349	1.21	10.5%
20	974	4.33	6.7%
30	10513	67.9	3.8%

Outline

Recapitulation

Recapitulation

- We proposed to search for good lower bounds on the Two-Stage Stochastic Knapsack Problem with Probabilistic Constraint using a branch-and-bound algorithm

Recapitulation

- We proposed to search for good lower bounds on the Two-Stage Stochastic Knapsack Problem with Probabilistic Constraint using a branch-and-bound algorithm
- Upper bounds on the overall problem as well as on subproblems can be obtained by solving the corresponding relaxed problem

Recapitulation

- We proposed to search for good lower bounds on the Two-Stage Stochastic Knapsack Problem with Probabilistic Constraint using a branch-and-bound algorithm
- Upper bounds on the overall problem as well as on subproblems can be obtained by solving the corresponding relaxed problem
- The relaxed problem can be solved using a stochastic gradient algorithm

Recapitulation

- We proposed to search for good lower bounds on the Two-Stage Stochastic Knapsack Problem with Probabilistic Constraint using a branch-and-bound algorithm
- Upper bounds on the overall problem as well as on subproblems can be obtained by solving the corresponding relaxed problem
- The relaxed problem can be solved using a stochastic gradient algorithm
- Lower bounds are obtained during the b-a-b algorithm by bounding the second stage problem from above

Thank you!

Merci!

Dank(e)!