Towards an Ant Colony Optimization algorithm for the Two-Stage Knapsack problem

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1 The Two-Stage Knapsack Problem

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Outline

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The Deterministic Knapsack Problem

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Maximize the total reward of chosen items whose total weight respect knapsack capacity.

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Applications

Logistics - Resource allocation - Scheduling - Network Optimization etc.

The **Stochastic** Knapsack Problem **with Random Weights**

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Question

How to handle the fact that chosen items might not respect knapsack capacity?

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- $\mathbb{P}\{\chi = \chi^k\} = p^k$

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- Vacant beds filled with last minute offers

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$$(TSKP) \quad \max_{x\in\{0,1\}^n} \sum_{i=1}^{n} r_i x_i$$

s.t.

x: decision vector of 1st stage

Two-Stage Knapsack Problem

$$(TSKP) \quad \max_{x \in \{0,1\}^n} \sum_{i=1}^{n} r_i x_i + \mathbb{E}[\mathcal{Q}(\mathbf{x}, \chi)]$$

$$\text{s.t.} \quad \mathcal{Q}(x, \chi) = \max_{\mathbf{y}^+, \mathbf{y}^- \in \{0,1\}^n} \sum_{\mathbf{i}=\mathbf{1}}^{\mathbf{n}} \bar{\mathbf{r}}_\mathbf{i} \mathbf{y}_\mathbf{i}^+ - \sum_{\mathbf{i}=\mathbf{1}}^{\mathbf{n}} \mathbf{d}_\mathbf{i} \mathbf{y}_\mathbf{i}^-,$$

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Outline

Natural idea: try metaheuristics!

But why an **ACO**-metaheuristic?

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Heuristic utility measure I: Problems

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Heuristic utility measure II: Propositions

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$\mathcal{K}_i$: set of scenarios where item i still fits

- Simple utility measure:

$$\eta_i^S = \sum_{k \in \mathcal{K}_i} p^k \frac{r_i}{\chi_i^k}$$

Heuristic utility measure II: Propositions

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Heuristic utility measure II: Non-utility measures

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$$\nu_i^S = \sum_{k \notin \mathcal{K}_i} p^k \frac{d_i}{\chi_i^k} \qquad \nu_i^S = \sum_{k=1}^{K} p^k \frac{\bar{r}_i}{\chi_i^k}$$

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Heuristic utility measure II: Non-utility measures

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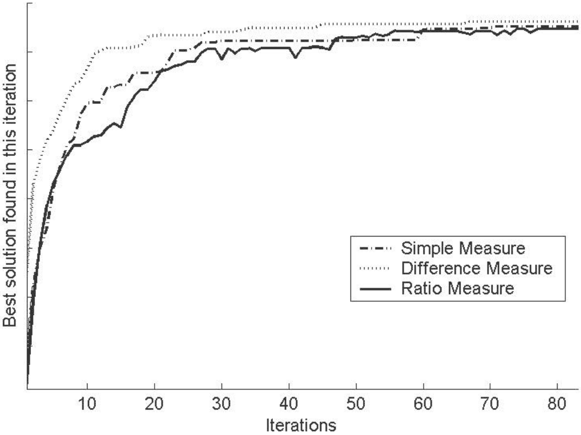
- Simple non-utility measure:
  $$\nu_i^{S_1} = \sum_{k \notin \mathcal{K}_i} p^k \frac{d_i}{\chi_i^k} \quad \text{or} \quad \nu_i^{S_2} = \sum_{k=1}^{K} p^k \frac{\bar{r}_i}{\chi_i^k}$$
- Difference non-utility measure:
  $$\nu_i^D = \sum_{k \notin \mathcal{K}_i} p^k \frac{d_i - r_i}{\chi_i^k}$$
- Ratio non-utility measure:
  $$\nu_i^R = \sum_{k \notin \mathcal{K}_i} p^k \frac{d_i / r_i}{\chi_i^k}$$

Utility of termination:

$$\eta_{n+1} = min_{i \in \{1,\dots,n\}} \nu_i$$

Outline

Comparison of the different utility measures I



Comparison of the different utility measures II

		Difference measure			Simple measure		
n-K-t	Inst.	Runs	Gap	CPU(s)	Runs	Gap	CPU(s)
100-5-0.25	3/3	57%	0.02%	35	13%	0.05%	30
100-5-0.5	2/3	28%	0.01%	57	1%	0.03%	52
100-5-0.75	1/3	1%	0.02%	69	0%	0.02%	71
100-10-0.25	3/3	93%	0.06%	47	63%	0.01%	34
100-10-0.5	2/3	23%	0.01%	72	0%	0.03%	63
100-10-0.75	1/3	15%	0.02%	85	0%	0.04%	85
100-30-0.25	2/3	58%	0.02%	147	0%	0.12%	107
100-30-0.5	3/3	63%	0.01%	232	8%	0.02%	179
100-30-0.75	1/3	25%	0.01%	295	0%	0.03%	183

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- **Comparison with other metaheuristics**

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Thank you!

:-)

Obrigada!