Stochastic Optimization
IDA PhD course 2011ht

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5. Lecture: Two-Stage Problems
10. November 2011

1 Randomness occurs in the constraint function

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Outline

Deterministic Opt. Model → Stochastic Programming Model

$$\max_{x \in X} \quad f(x) \qquad\qquad \min_{x \in X} \quad f(x)$$

$$\text{s.t.} \quad G(x) \leq 0 \qquad \rightarrow \qquad \text{s.t.} \quad G(x, \chi) \leq 0$$

$\chi \in \Omega \subseteq \mathbb{R}^s$: random vector
$G : \mathbb{R}^n \times \mathbb{R}^s \rightarrow \mathbb{R}^m$
$G(x, \chi) = (g_1(x, \chi), \ldots, g_m(x, \chi))$

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Question

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- Average violation not too bad → simple recourse ✓

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- Average violation not too bad → simple recourse ✓
- Feasible after correction has been made → multi-stage

Outline

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Basic Idea(s)

- Allow violation
- Restrict average "amount of violation"
- Introduce penalty per "violation unit"
- Find good trade-off: cost $\leftrightarrow$ penalty

Simple-Recourse problem

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$$\min_{x \in X} \quad f(x)$$

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$$\min_{x \in X} \quad f(x) + \sum_{i=1}^{m} d_i \cdot \mathbb{E}\left[[g_i(x,\chi)]^+\right]$$

- $d_i > 0 \ \forall i \in \{1, \ldots, m\}$

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- $[x]^+ = max(0, x)$

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Then the following holds:

$$g(\cdot,\hat{\chi}) \text{ is convex } \forall \hat{\chi} \implies \gamma \text{ is convex}$$

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2. *For all* $x \in \mathbb{R}^n$ $\gamma(x) < \infty$.

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5. $a(\chi)x - b$ *discretely distributed* $\Rightarrow \gamma$ *is piecewise linear.*

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- Feasibility in all scenarios: Conservative solutions
- Chance-Constraints: Infeasibility of solution possible
- Simple Recourse: No actual "recourse" action
- In the end: feasible solution $\rightarrow$ most realistic?
- Costs/rewards of recourse action taken into account

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Basic structure of Two-Stage Programming Problem

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6. Make corrective decision ("recourse"):
   - $\rightarrow$ Make solution feasible
   - $\rightarrow$ Decrease total cost

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General Two-Stage Problem

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$$\min_{x \in X} \quad f^1(x)$$

s.t.

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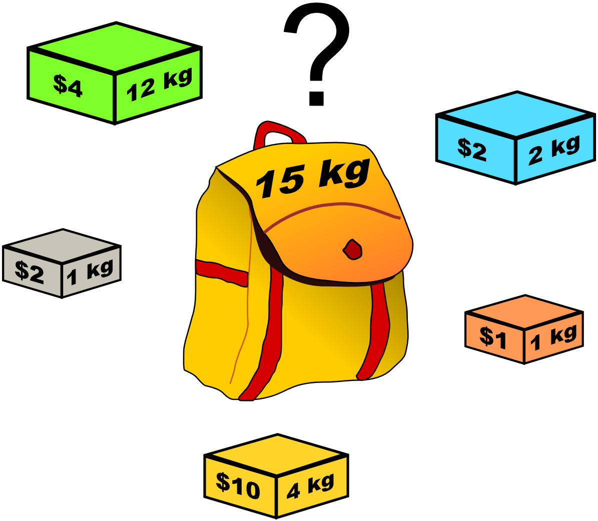
Outline

Example: TSKP



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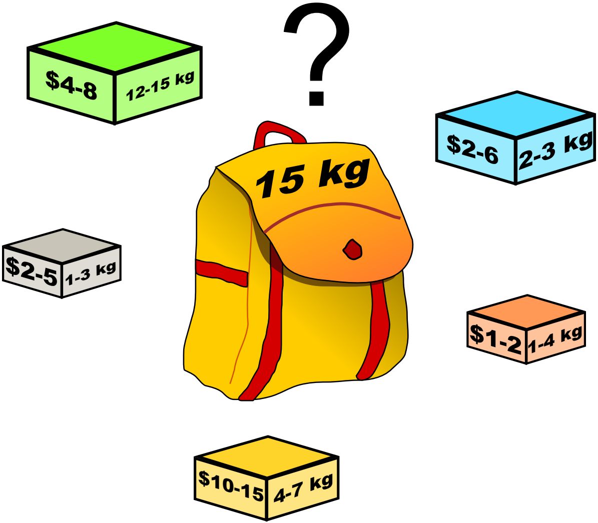

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Two-Stage Stochastic Knapsack problem

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  ...have to be removed in case of an overweight
  ...can be added if capacity sufficient
  ...can be exchanged to increase gain.

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- Correction of the decision causes penalty

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Application: Travel Agency

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- Knapsack $\simeq$ Hotel Complex
- Weight capacity $\simeq$ Total number of beds
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- Vacant beds filled with last minute offers

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s.t.

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$T(\chi)$: Technology matrix
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Outline

Linear Simple-Recourse problem

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$$\min_{x \in X} \quad c^T x + \sum_{i=1}^{m} d_i \cdot \mathbb{E}\left[[h_i(\chi) - T_i(\chi)x]^+\right]$$

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Equivalent formulation in two stages

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$\mathbb{I}_n$: Identity matrix of dimension n

Simple Recourse Problem formulated as Two-Stage Problem

- No first stage constraints.

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Deterministically Reformulated Linear Two-Stage Problem

$$\min_{x\geq 0} \quad c^T x +$$

$$\text{s.t.} \quad Ax \geq b, \tag{8a}$$

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Outline

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Given: linear two-stage problem
Let:

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  ($\rightarrow$ second-stage problem primal and dual feasible)

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- *subdifferentiable in* x*,*
- *a polyhedral convex function.*

QUESTIONS?

What about next week?