Stochastic Optimization
IDA PhD course 2011ht

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10. Lecture: Computational examples
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Outline

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Is-Situation

- Use average/expecteted values
- No adaptations to current situation

Disadvantages

- Average gain $\neq$ computed expected value
- No flexibility
- Solution not robust
- Might entail infeasible solutions

Solutions

- Online Optimization ?
- Robust Optimization
- Stochastic Programming
- ...

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Typical application fields

- Capacity planning
- Energy sector
- Finance
- Forestry
- Military
- Production / Supply chain
- Scheduling
- Transportation (of humans, goods,...)
- Water management
- ...

Outline

Example: Continuous Knapsack with uncertain capacity

$$\max_{x\in[0,1]^3} \quad c^T x$$
$$\text{s.t.} \quad a^T x \leq \chi$$

- $c = (10, 15, 20)$
- $a = (5, 10, 20)$
- $p_1 = \mathbb{P}\{\chi = \chi_1\} = \mathbb{P}\{\chi = 3\} = 0.2$
- $p_2 = \mathbb{P}\{\chi = \chi_2\} = \mathbb{P}\{\chi = 12\} = 0.3$
- $p_3 = \mathbb{P}\{\chi = \chi_3\} = \mathbb{P}\{\chi = 25\} = 0.5$

Scenario dependent solutions

$$\max_{x\in[0,1]^3} \quad c^T x(\chi)$$
$$\text{s.t.} \quad a^T x(\chi) \leq \chi$$

- $x(3) = (0.6; 0; 0)^T$; $z(3) = 6$
- $x(12) = (1; 0.7; 0)^T$; $z(12) = 20.5$
- $x(25) = (1; 1; 0.5)^T$; $z(25) = 35$
- $z = \mathbb{E}[z(\chi)] = \sum_{i=1}^{3} p_i z(\chi) = 24.85$

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Average value approach

$$\max_{x\in[0,1]^3} \quad c^T x$$
$$\text{s.t.} \quad a^T x \leq \mathbb{E}[\chi]$$

- $x = (1; 1; 0.085)^T$
- $z = 26.7$

Worst case approach

$$\max_{x \in [0,1]^3} \quad c^T x$$
$$\text{s.t.} \quad a^T x \leq \inf_{\chi \in \Omega} \chi$$

- $x = (0.6; 0; 0)^T$
- $z = 6$

Chance Constrained Approach

$$\max_{x \in [0,1]^3} \quad c^T x$$
$$\text{s.t.} \quad \mathbb{P}\{a^T x \leq \chi\} \geq \alpha$$

- $\mathbb{P}\{a^T x \leq \chi\} \geq 0.8 \Leftrightarrow a^T x \leq 12$
- $x = (1; 0.7; 0)^T$
- $z = 20.5$

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Two-Stage Approach

$$\max_{x\in[0,1]^3} \quad c^T x - \sum_{i=1}^{3} p_i q(\chi_i) y(\chi_i)$$
$$\text{s.t.} \quad a^T x - y(\chi_i) \le \chi_i \quad \forall i$$

- $q(\chi_1) = 2;\ q(\chi_2) = 3;\ q(\chi_3) = 6$
- $x = (1; 1; 0)^T$
- $y(\chi_1) = 12$
- $y(\chi_2) = 3$
- $y(\chi_3) = 0$
- $z = 17.5$

Simple Recourse Approach

$$\max_{x \in [0,1]^3} \quad c^T x - \mathbb{E}[q(\chi)[a^T x - \chi]^+]$$

- $z = 17.5$

Outline

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Reference

J. Balasubramanian and I. E. Grossmann
Approximation to Multistage Stochastic Optimization in Multiperiod Batch Plant Scheduling under Demand Uncertainty. (2003)
http://egon.cheme.cmu.edu/Papers/BalasubMultistage.pdf

Multiproduct, multiperiod Batch production

- Different products
- Different stages
- Different costs (materials, stockage/holding, under-production...)

$\Rightarrow$ Hard scheduling problem

Uncertainties

- Demand
- Processing times
- Costs

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Online Optimization?

- Long processing times
- Need to make predictions
- Holding/storage costs $\leftrightarrow$ idle costs
- Difficult mathematical problem $\leftrightarrow$ reaction times

Example Problem

- single-stage single-unit bath plant
- 2 products
- 3 processing modes (batch sizes)
- Uncertain demand
- 2 time periods (same demand distribution)

Product	**Batch size** (tons)	**Proc. time**	**REV** ($/tons)	**XC** ($/tons)	**LC** ($/tons)
A	0-5 5-10 10-25	2 4 6	100	10	20
B	0-5 5-10 10-25	3 5 7	250	20	50

Event	**Probability**	**Demands (tons)**
1	0.25	A:10, B:0
2	0.75	A:20, B:5

Deterministic approach

- Work with expected demands over entire horizon
- Model predicts: $5375
- Actual expected profit: $4559

Two-Stage approach

- First-stage: production scheduling over both time periods
- Second-stage: Amount to be sold / Unsatisfied amount
- Expected profit: $5275 (+15%)

Three-Stage approach

- First-stage: production scheduling over 1. period
- Second-stage: production scheduling over 2. period
- Third-stage: Amount to be sold / Unsatisfied amount
- Expected profit: \$5325 (+17%)

Idea

- Approximate solution
- Shrinking time horizon strategy
- Basic structure
  - Solve two-stage problem over remaining time horizon (given past demands)
  - Implement solution *only* in next time period

!

- Problem solved in advance! ($\neq$ online optimization)
- Two-Stage problem for all nodes in scenario tree

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1. Numerical Example

- single product
- 3 stages
- 3 processing modes each
- 5 time periods (different probability distributions)

Approach	Expected revenue (1000$)	+	CPU time (sec)
Det.	656.23	0%	3
2-stage	672.04	2%	20
SH	717.32	9%	360
6-stage	722.43	10%	>50,000

2. Numerical Example

- 4 products
- 8 tasks
- 6 processing units
- 3 processing modes for each combination
- 3 time periods (different probability distributions)

Approach	**Expected revenue ($)**	$+$	**CPU time (sec)**
Det.	59,573	0%	10
2-stage	62,945	6%	35
SH	75,452	27%	180
4-stage	75,851	27%	$>$100,000

Conclusion

- Working with average values $\rightarrow$ false estimations
- Choosing two/multi-stage model: important increase in gain
- Approximations help to decrease CPU time
- Approximations still much better than det. model

QUESTIONS?

Tack sa mycket!