Introduction to Stochastic Combinatorial Optimization

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Guest Lecture at the CUGS PhD course
"**Heuristic Algorithms for Combinatorial Optimization Problems**"

"Facts"

Combinatorial "real world problems" often subject to uncertainties

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- Own decision depends on future decision of other parties: competition, clients, government...
- Setting of problem might change: weather, location...

What is the interest of Stochastic Combinatorial Optimization?

Combinatorial "real world problems" often subject to uncertainties

- Not all parameters known when decision has to be made: market fluctuations, available capacity...
- Own decision depends on future decision of other parties: competition, clients, government...
- Setting of problem might change: weather, location...

Definition

Stochastic Combinatorial Optimization concerns the study and resolution of Combinatorial Optimization problems that involve uncertainties.

Objectives of this lecture

- Give you examples of SCO-problems.

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- Give you an idea of how SCO-problems can be solved.

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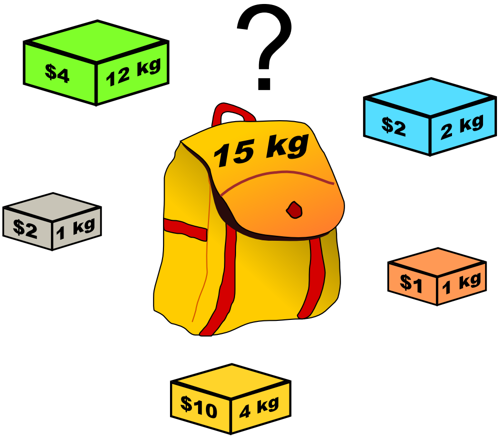
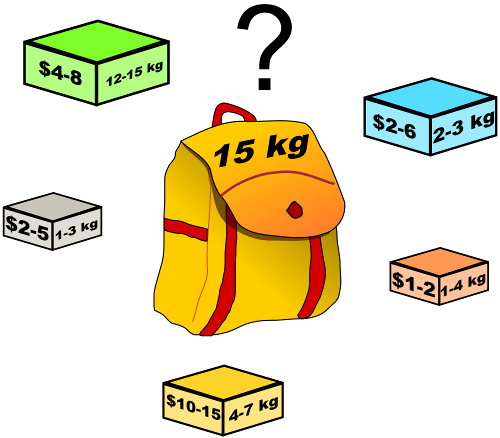
- Give you examples of SCO-problems.
- Give you an idea of how uncertainties can be modeled (most common models).
- Give you an idea of why Stochastic Optimization is hard.
- Give you an idea of how SCO-problems can be solved.
- Give you an idea of why metaheuristics are important tools in SCO.

Outline

Deterministic Knapsack problem



Stochastic Knapsack problem



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- knapsack constraint violated $\Rightarrow$ penalty

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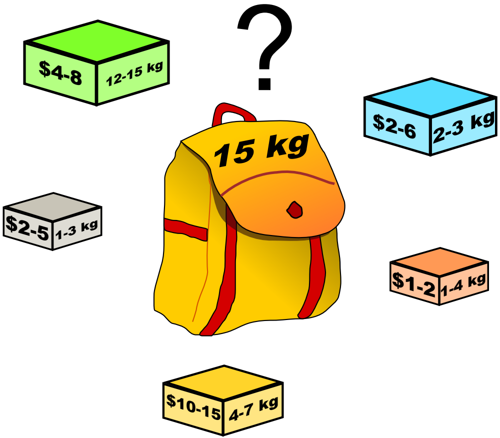

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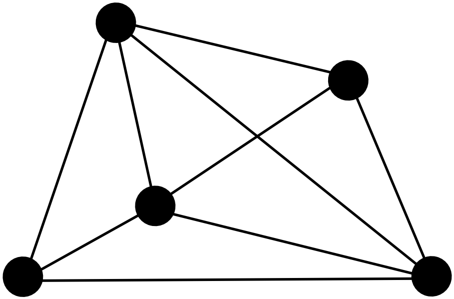
Possible ways to handle capacity constraint

- knapsack constraint violated $\Rightarrow$ penalty
- probability of capacity violation restricted
- decision can be corrected later (add. costs/reduced rewards)

Deterministic Graph Coloring



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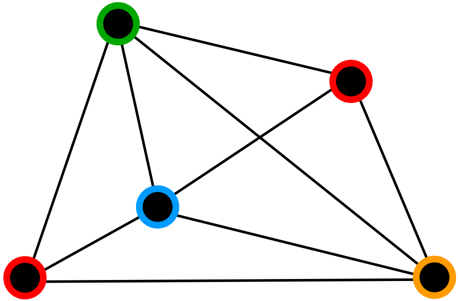

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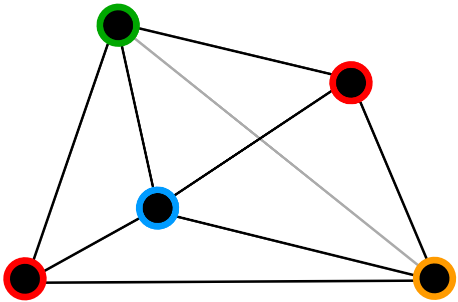

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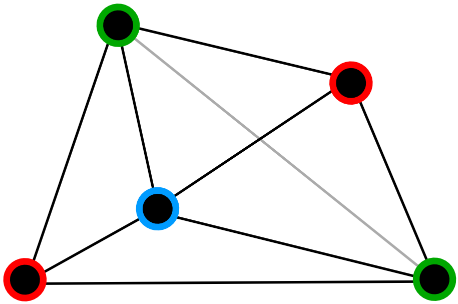

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- "cost" of colors random

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Deterministic CO Model → Stochastic CO Model

$$\max_{x \in \{0,1\}^n} \quad f(x)$$
$$\text{s.t.} \quad g(x) \leq 0$$

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$\chi \in \Omega \subseteq \mathbb{R}^s$: vector with random entries

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- Good result "on average"
- Objective function can often be reformulated deterministically

Disadvantages:

- We might encounter very "bad cases"

Minimize variance

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Disadvantages:

- Makes not much sense without benchmark for expected costs

Minimize variance

$$\min_{x\in\{0,1\}^n} \quad \lambda Var\left[f(x,\chi)\right] + \mathbb{E}\left[f(x,\chi)\right]$$
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$$\min_{x \in \{0,1\}^n} \quad Var\left[f(x,\chi)\right]^{\lambda} \cdot \mathbb{E}\left[f(x,\chi)\right]$$
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- Worst case might be very improbable
- Average might be high

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- Constraint forced to be satisfied in even very improbable cases

Chance-Constrained model

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Disadvantages:

- No restriction of "magnitude" of allowed violation
- What happens if constraint is violated?

Simple-Recourse model

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Advantages:

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Disadvantages:

- Probability of violation not restricted

Two-Stage model

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$$\min_{x \in \{0,1\}^{n_1}} \quad f(x) + \mathbb{E}\left[Q(x,\chi)\right]$$

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  - $\rightarrow$ Second-stage problem $\mathcal{NP}$-hard

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$$\begin{aligned} \min_{x \in \{0,1\}^{n_1}} \quad & f(x) + \mathbb{E}\left[Q(x,\chi)\right] \\ \text{s.t.} \quad & Q(x,\chi) = \min_{y \in \{0,1\}^{n_2}} \quad \bar{f}(y) \\ & \qquad\qquad\qquad \text{s.t.} \quad g(x,y,\chi) \leq 0 \end{aligned}$$

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- High number of binary decision variables and constraints

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Problem

Generally only possible under assumption of special distributions!

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K outcomes $\chi^1, \ldots, \chi^K$ with prob.'s $p^1, \ldots, p^K$

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$$\sum_{i=1}^{n} \chi_i^k x_i \leq c + M z_k \quad \forall\, k = 1, \ldots, K,$$

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- To approximate original problem high number of samples might be needed

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